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ABSTRACT

Designed for the students who have taken algebra and geometry and who need to strengthen their skills in problem solving and mathematical applications, this guidebook on minimum course content emphasizes the kinds of skills and procedures used in college placement tests. Overall course goals are specified, a course cutline is provided, performance objectives are listed, and references keyed to the performance objectives are provided. Also included is a sample test with an answer key. (JP)



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AUTHORIZED COURSE OF INSTRUCTION FOR THE



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DIVISION OF INSTRUCTION-1973

QUINWESTER MATHEMATICS COURSE OF STUDY FOR

MATH REVIEW 5265.01

Written by
Lynda H. Kramer

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
1971-72



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PREFACE

The following course of study has been designed to set a <u>minimum standard</u> for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.



CATALOGUE DESCRIPTION

A course designed for the students who have taken Algebra and Geometry and who need to strengthen their skills in problem solving and mathematics applications. Emphasis will be on the kinds of skills and procedures used in the college placement tests (SCAT, PSAT, and CEEB).

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OVERALL GOALS

The student will:

- 1. Become familiar with the types, forms and methods of presentation of questions on the mathematics section of PSAT, SAT, and CEEB examinations.
- 2. Improve his skill and speed in answering standard questions.
- 3. Improve his skill in analyzing and sythesizing general information given in questions in which reasoning, judgment, comparison, and evaluations are called for.
- 4. Provide a measure of his own areas of weakness and strength by taking sample test items provided.
- 5. Have an opportunity to synthesize computational analytical skills in direct application in problem settings.

PERFORMANCE OBJECTIVES

The student will:

- 1. Perform computation skills with non-negative rational numbers.
- 2. Solve word problems involving non-negative rationals.
- 3. Perform computational skills with rational numbers, rational expressions, and radicals.
- 4. Solve first and second degree number sentences in one variable and first degree number sentences in two variables.
- 5. Graph first and second degree number sentences.
- 6. Solve word problems using techniques of algebra.
- 7. Demonstrate an understanding of elementary relationships and theorems of geometry.
- 8. Solve word problems involving geometric applications.



KEY TO REFERENCES

- Brownstein, Samuel and Weiner, Mitchel. How to Prepare for College Entrance Examinations. Woodbury, New York: Barron's Educational Series, Inc., 1970.
- Rich, Barnett. <u>Mathematics for the College Boards</u>. New York: School Publications, Inc., 1970.
- Turner, David R. Scholastic Aptitude Tests. New York. Arco, 1969.

SOURCES FOR PROBLEMS

- Watkin, Harold. How to Pass College Board Admissions. New York: Crowles Educational Corporation, 1970.
- Wechsler, Louis; Blum, Martin; and Friedman, Sidney.

 <u>College Entrance Examinations</u>. New York: Barnes and Noble, 1971.



OUTLINES, STRATEGIES, AND REFERENCES

- Arithmetic Skills
 - A. Fractions
 - 1. Meaning of fractions
 - a. Part of a whole number
 - b. Ratio
 - c. Division
 - 2. Fractional numbers
 - a. Written as improper fractions
 - b. Written as mixed numbers
 - 3. Equivalent fractions
 - a. Higher terms
 - b. Simplest form
 - 4. Lowest Common Demominator
 - a. By inspection
 - b. By prime factorization of the denominator
 - 5. Operations
 - a. Addition
 - b. Subtraction
 - c. Multiplication
 - d. Division
 - e. Combination of operations
 - 6. Comparing fractions
 - a. Common denominator
 - b. Proportion
 - B. Decimals
 - 1. Reading and writing



- 2. Operations
 - a. Addition
 - b. Subtraction
 - c. Multiplication
 - d. Division
 - e. Multiplication/division by powers of 10
- Conversions
 - a. Decimals to fractions
 - b. Fractions to decimals
- C. Percent
 - 1. Meaning
 - 2. Conversions
 - a. Percent to decimal and decimal to percent
 - b. Percent to fraction and fraction to percent
 - 3. Three cases of percent
 - a. What percent of ___ is ___?
 - b. Find ___% of ____.
 - c. is _ % of what number?

STRATEGIES

- 1. To improve the student's computation skills, work for understanding and then for shortcuts in the processes.
- 2. In multiplying fractions or mixed numbers, be sure the student learns the "cancellation" method, but stress that it is used only when multiplication of factors is involved.
- 3. Stress proportions since they are a valuable tool in the solution of many problems.
- 4. Make the student aware of the formula $\% = \frac{is}{of}$ if he is not already familiar with it.



REFERENCES

| | (1) | (2) | (3) |
|-----------|---------|-------|---------|
| Fractions | 228-236 | 18-22 | 273-277 |
| Decimals | 240-242 | 26-28 | 279-282 |
| Percents | 242-249 | 32-45 | 284-287 |

II. Arithmetic Applications

- A. Problems Involving Fractions
- B. Problems Involving Decimals
- C. Problems Involving Proportion
- D. Problems Involving Percent
 - 1. Commission
 - 2. Discount
 - 3. Increase and decrease
 - 4. Profit and loss
 - 5. Interest
 - a. Simple
 - b. Compound
 - 6. Taxation

STRATEGIES

- 1. As much time as possible should be spent on problem solving as this is usually the student's weakest area.
- 2. Insist that students draw diagrams whenever possible to assist them in solving problems.
- 3. Have students write out a method for doing the types of problems with which they have difficulty. This will help them organize their work and it will provide a reference which they can review from time to time.
- 4. Understanding vocabulary is an important part of problem solving, so be sure to discuss the meaning of words often used in problems. This is particularly true in the applications of percent.



REFERENCES

| | (1) | (2) | (3) |
|-----------------------|---------|---------|----------|
| Percentage | | 32-45 | 285-287 |
| Fraction and Mixture | 261-263 | 134-144 | 288-291 |
| Profit and Loss | | 28-31 | 292-297 |
| Interest and Taxation | | 152-157 | 298-306 |
| Ratio and Proportion | | 33-39 | -307-312 |

III. Algebra Skills

A. Real Numbers

- 1. Properties
- 2. Computation with signed numbers

B. Laws of Exponents

- 1. Products
- 2. Quotients
- 3. Powers of powers

C. Polynomials

- 1. Addition
- 2. Subtraction3. Multiplication
- 4. Division
- 5. Factoring

D. Rational Expressions

- 1. Addition
- 2. Subtraction
- 3. Multiplication
- 4. Division
- 5. Simplification

E. Expressions with Radicals

- 1. Addition
- 2. Subtraction
- 3. Multiplication4. Division
- 5. Simplification



- F. Solving Number Sentences
 - 1. Linear Equations
 - 2. Quadratic equations
 - 3. Simultaneous linear equations
 - 4. Inequalities
- G. Graphing Number Sentences
 - 1. Linear
 - 2. Quadratic

STRATEGIES

- 1. When reviewing rational expressions, compare each operation to the parallel operation with fractions. The simplification of the expressions from rational expressions to fractions makes the properties and processes much easier to understand.
- 2. In reviewing the solution of number sentences, develop sets of exercises that include all types of solutions in one assignment. This gives the student a chance to review each process and also decide which process should be used.
- 3. In reviewing graphing, have students practice giving information for a given graph. Since they will not usually be required to graw a graph, this skill does not need to be emphasized.

REFERENCES

| | (1) | (2) | (3) |
|-----------------------------|---------|----------------|---------|
| Basic Operational Skills | 220-225 | 62 - 68 | 343-351 |

- IV. Algebra Applications
 - A. Special Types of Problems
 - 1. Work
 - 2. Rate, time, distance
 - 3. Mixture
 - 4. Age
 - 5. Coin
 - 6. Two-digit numeral
 - B. Miscellaneous Types of Problems



STRATEGIES

- Work problems receive heavy emphasis on standardized tests. Some time should be spent analyzing the different types of work problems and formulating patterns for solving.
- 2. Be sure students have a system for approaching the solution of problems. This should include the drawing of diagrams where possible, as well as defining the variables they use.
- 3. Stress the importance of checking the solution with the original problem to see if the answer is reasonable.

REFERENCES

| | (1) | (2) |
|-------------------------|---------|---------|
| Annliantion | • | 19-70 |
| Application Problems | 226-228 | 123-134 |

V. Geometry

- A. Geometric Figures
 - 1. Names
 - 2. Properties
 - 3. Associated parts
- B. Relationships of Geometric Figures
 - l. Angles
 - 2. Perpendicular and parallel lines and planes
 - 3. Triangles
 - 4. Other polygons
 - 5. Circles

STRATEGIES

- 1. It might be worthwhile to have students list the more important relationships of geometric figures. This would be a review in itself, and would provide a ready reference as they worked through problem sets.
- 2. Be sure the student can sketch the geometric figures and their parts. Practice can be included in the daily work by requiring the student to draw figures for each problem.



REFERENCES

218-240 267-276 246-314 Plane Geometry

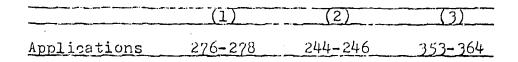
- VI. Geometric Applications
 - A. Special Types of Problems
 - 1. Perimeter
 - 2. Area

 - 3. Volume4. Pythagorean theorem
 - 5. Similar figures proportion
 - b. Miscellaneous Types of Problems

STRATEGIES

- 1. It would be helpful to have the student learn some of the Pythagorean triples in addition to the (3,4,5) Pythagorean triple.
- 2. Be sure the student knows the common formulas for area, surface area, and volume.
- 3. Many applications involve proportions resulting from similar figures. Review the conditions for similarity, and the selection of corresponding parts of similar figures.

REFERENCES





SAMPLE TEST ITEMS

I. Arithmetic Skills

| | PROBLEMS | ANSWERS |
|-----|--|----------------------------------|
| 1. | Find the LCD of $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{6}$, and $\frac{1}{10}$. | 840 |
| 2. | Add $16\frac{3}{8}$, $4\frac{4}{5}$, $12\frac{3}{4}$, and $23\frac{5}{6}$. | 57 <u>91</u> 120 |
| 3• | Subtract $27\frac{5}{14}$ from 43 $\frac{1}{6}$ ° | 15 17 21 |
| 4. | Multiply 17 $\frac{5}{8}$ by 128. | 2256 |
| 5• | Divide $\frac{5}{8}$ by $\frac{3}{8}$. | $\frac{20}{9}$ or $2\frac{2}{9}$ |
| 6. | Add 37.03, 11.5627, and 3.4005. | 51.9932 |
| 7. | Subtract 4.64324 from 7. | 2.35676 |
| 8. | Multiply 27.3 by 16.943. | 462.5439 |
| 9• | 19.6 divided by 3.2 carried to three decimal places is? | 6.125 |
| 10. | Convert $\frac{5}{11}$ to decimal form. (nearest hundred | th) <u>.45</u> |
| 11. | What is $64 \frac{2}{3}$ in fractional form? | 194 |
| 12. | In decimal notation find the difference between $\frac{9}{8}$ and $\frac{3}{5}$. | |
| 13. | What percent is 2 $\frac{1}{2}$? | 2 50% |
| 14. | Find the fractional equivalent of 5.4%. | <u>27</u> 500 |
| 15. | Convert $\frac{3}{4}\%$ to decimal notation. | .0075 |



Arithmetic Skills (continued)

16. Convert 2 $\frac{3}{7}$ % to fractional form.

<u>17</u> 700

17. Find $4\frac{1}{2}\%$ of 890.

40.05

18. What percent of 39 is 25? (nearest tenth)

64.1%

II. Arithmetic Applications

| | PROBLEMI. | ANSWERS |
|-----|--|---------------|
| 1, | The entrance price to see an exhibition was reduced by 25%, but the dully attendance increased 30%. What was the effect of such a price reduction on the dully receipts. | 2 ½% decrease |
| 2. | A mar owned 50 shares of stock worth \$75 each. The firm declared a dividend of 4% payable in stock. How many shares did he own AFTER the stock dividend? | _52 |
| 3. | A man spent $\frac{15}{16}$ of his entire fortune to buy | |
| | a house for \$7500. How much was his original fortune? | \$8,000 |
| 4. | What percent of $\frac{5}{6}$ is $\frac{3}{4}$? | 90% |
| 5. | A typist earned \$1350 in a given year which was $12\frac{1}{2}\%$ more than she earned the year before. How much did she earn the year before? | \$1,200 |
| 6. | A certain pole casts a shadow 24 feet long. At the same time another pole 3 feet high casts a shadow 4 feet long. How high is the first pole? | 18 feet |
| 7. | A man purchased an equal number of \$3, \$2, and 75¢ ties. He spent \$40.25 for all of the ties. How many of each did he buy? | 2 |
| 8. | If the entrance requirement of a certain college is 82, what mark must a student have in Geometry (weight 2) to be able to enter if his other marks are: English 88 (weight 3); Spanish 78 (weight 2); and History 80 (weight 2) | 2)? <u>79</u> |
| 9• | Find the interest on \$480 at $3\frac{1}{6}\%$ for two months and 15 days. | \$3.50 |
| 10. | What is the premium on a \$7200 policy at \$10.67 per 100? | \$48.24 |



III. Algebra Skills

PROBLEMS

 $\frac{1}{2x^2}$

ANSWERS

1. Reduce to lowerst terms: $\frac{3x}{6x^3}$ 2. Multiply $3x^2$ and $4x^5$

- 12x⁷
- 3. The roots of the equation $x^2 + 16 = 0$ are: (1) real and equal (2) real and unequal; or (3) imaginary.
- (3) imaginary
- 4. The sum of the roots of the quadratic equation $2x^2 4x 5 = 0$ is?
- 2
- 5. Solve for x in the equation $\sqrt{x+2} = 3$
- 7
- 6. The positive root of the equation $2x^2$ 3x 2 = 0 is?
- 2
- 7. The roots of the equation $2x^2 8x + 3 = 0$ are: (1) unequal; (2) irrational; or (3) both
- (3) both 23 -8

8. Simplify $(\frac{5}{2} - 3) \cdot \left| -6 + \frac{1}{4} \right|$ 9. Factor $6x^2 - 24$

- 6(x + 2)(x 2)
- 10. Subtract $\frac{t}{r^2 + 2r 3}$ from $\frac{4t + 3}{r 1}$
- $\frac{4rt + 11t + 3r + 9}{(r 1)(r + 3)}$

11. Simplify: $12 + \frac{2\sqrt{3}}{3}$

12. Solve: $\begin{cases} y + x = 4 \\ 3x - 2y = 1 \end{cases}$

 $\left(\frac{9}{5}, \frac{11}{5}\right)$

PROBLEMS

ANSWERS

| 1. | If $\frac{1}{4}$ of A's morey is equal to $\frac{2}{7}$ of B's | |
|----|---|-----------|
| | money and their total wealth is \$75.00, how much money has B? | 3500 |
| 2. | An estate is divided among three heirs, A,B, and C, so that A has $\frac{5}{12}$ of the whole estate, and B has twice as much as C. A has 56 acres more than C. How many acres in the entire estate? | 252 acres |
| 3• | If x varies directly as y^2 and if $x = 9$ when $y = 2$, what is the value of x when $y = 8$? | 144 |
| 4. | If clerk A can type 50 letters in 10 minutes, whereas clerk B can only type 40 letters in 10 minutes, in how many minutes can they type 360 letters together? | 40 |
| 5• | If in 5 days a clerk can copy 125 pages, with 36 lines on each page and 11 words to the line, how many pages of 30 lines each and 12 words to the line can be copy in 6 days? | 165 |
| 6. | A can do a job in 4 days, B assists him for 2 days and then both stop working. C who can do the work in 10 days, works 4 days and completes the job. How long would it take B to do the complete job himself? | 20 days |
| 7• | A man can cover a distance of 360 miles in 12 hours by automobile and 30 hours on foct. He starts out in his car but it breaks down on the way, and he walks the rest of the distance spending twice as much time on foot as in the automobile. How long did the trip take? | 20 hours |
| 8 | Two trains running on the same track travel at the rates of 25 and 30 mph. If the first train starts out an hour earlier, how long will it take the second train to catch up with it? | 5 hours |

Algebra Applications (continued)

PROBLEMS

ANSWERS

9. Two ships are 1550 miles apart and sailing towards each other. One sails at the rate of 85 miles per day and the other at the rate of 65 miles per day. How far apart will they be at the end of 9 days?

200 miles

10. A and B started toward one another at the same time from places 350 miles apart and met in 7 hours. If A's rate was 30 mph, what was B's rate?

20 mph



PROBLEMS ANSWERS

| J. • | In a parallelogram ABCD angle B is twice as large as angle A. How many degrees are there in angle A? | 600 |
|------|--|------|
| 2. | In a right triangle ABC, altitude CD is drawn to hypotenuse AB. If angle A contains 32° how many degrees are there in angle BCD? | 320 |
| 3• | How many degrees are there in each angle of a regular polygon of 8 sides? | 135° |
| 4. | Angle ABC formed by diameter AB and chord BC of a circle contains 30°. If the diameter of the circle is 10, find the chord AC. | _5 |
| 5• | A line parallel to base AB of triangle ABC intersects the sides AC and BC at points D and E respectively. If CD equals 4 and DA equals 6 and CE equals 8, find EB. | 1.2 |
| 6. | The area of a triungle is 24. If one side of the triangle is 12, find the altitude drawn to that side. | 4 |
| 7. | Find the area of a square circumscribed about a circle whose radius is 10. | 400 |
| 8. | If an acute angle of a parallelogram contains 73°, how many degrees are there in an obtuse angle of the parallelogram? | 107° |
| 9• | Find an altitude of an equilateral triangle whose side is 10. Express the answer in simple radical form. | 5√3 |
| 10. | The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments 2 and 8. Find the altitude. | 4 |
| 11. | The sum of the angles of a polygon is 5400. How many sides does the polygon have? | _5 |

Geometry (continued)

PROBLEMS 12. Find the length of the side of a rhombus whose diagonals are 8 and 6.

13. Tangents PA and PB from an external point P to circle 0 form an angle of 700. If radii OA and OB are drawn, how many degrees are there in angle AOB?

110°

ANSWERS

14. In a circle two parallel chords opposite side of the center have arcs of 100° and 120°. Find the number of degrees in one of the arcs included between the chords.

70°

VI. Geometry Applications

| | PROBLEMS | ANSWERS |
|----|--|--------------|
| 1. | An automobile wheel has a diameter of 24 inches. How many revolutions will it make to cover 3 miles? | 2520 approx. |
| 2. | A ladder 65 feet long is leaning against the wall. It's lower end is 25 feet away from the wall. How much further away will it be if the upper end is moved down 8 feet? | 14 feet |
| 3. | A square lot has a diagonal path cut through it. If the path is 40 yards long, what is the area of the lot? | 800 sq. yds. |
| 4. | How many boxes 3 inches by 4 inches by 5 inches can fit into a carton 3 feet by 4 feet by 5 feet? | 1728 |
| 5• | The area of a rectangular garden is 60 square yards and the width is 5 yards. How much longer is the perimeter than the sum of both diagonals? | 8 yards |
| 6. | What is the area of a right triangle whose hypotenuse is 5 inches and one of its legs is 3 inches? | 6 sq. inches |
| 7. | A box is 12 inches in width, 16 inches in length and 6 inches in height. How many square inches of paper would be required to cover it on all sides? | 720 sa. in. |

